
Exploring exact differential equations with GeoGebra and ChatGpt software

Abstract

In this paper, we will present the solutions of several exact differential equations obtained using the interactive software GeoGebra. Additionally, we will provide a concrete example of an exact equation solved simultaneously with the assistance of artificial intelligence (ChatGPT) and GeoGebra.

GeoGebra is included due to its significant motivational support in the teaching and modeling processes in mathematics, and ChatGPT is used to demonstrate the solutions it offers.

This educational support material is intended for universities, particularly for engineering students in differential equations and multivariable calculus courses.

Keywords: Dif. equation, GeoGebra, Engineering education, ChatGpt.

1 Introduction

These authors today discuss the advantages of using GeoGebra in the teaching and learning of mathematics. For instance, del Pino [1] highlights that this software is free, open-source, cost-effective for educational institutions, and simple, yet powerful. It features

a spreadsheet and various views that allow users to alternate between arithmetic, algebraic representations, and symbolic calculations. Additionally, Garcia [2] notes that GeoGebra is a technological resource that can be utilized in the learning process and should be included in lesson planning as didactic material, specifically for the development of activities, such as solving exact differential equations.

Based on these arguments, we propose to showcase different GeoGebra applets for some exact equations along with their solutions, which were used during the year 2024 in the differential equations course. This work aims to aid and enhance the teaching process of these topics, along with ChatGPT as a tool for self-learning in mathematics [3] for engineering students at the University of Antofagasta. This paper continues the project titled "Animations and Interactive Creations in First-Order Linear Differential Equations: The Case of GeoGebra" at the University of Antofagasta in Chile [4].

2 GeoGebra Applets of exact differential equations

Below we will show some examples of applets from GeoGebra of first-order linear partial differential equations and its solutions available in <https://www.geogebra.org/m/v4jttuxq>

Example 1

Be

$$(x^n - 1) dy + nyx^{n-1} dx = 0, \quad (2.1)$$

With $n = 3, 4, \dots, 13$,

and whose solution is given in blue by $x^n y - y = c$ in Figure 1, where c varies between -5 and 5 .

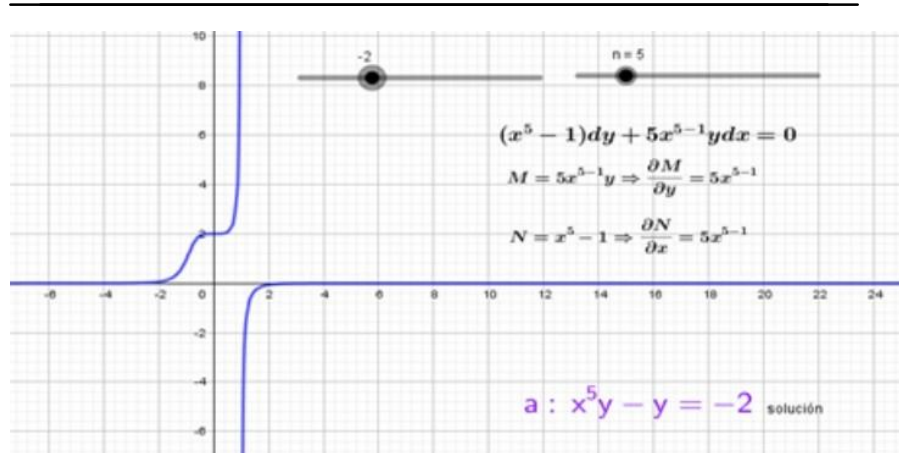


Figure 1

Example 2

Be

$$(x^n + y^n) dx + ny^{n-1}x dy = 0, \quad (2.2)$$

with $n = 3, \dots, 13$,

and whose solution is given in blue by $\frac{x^{n+1}}{n+1} + xy^n = c$ in Figure 2, where c varies between -5 and 5 .

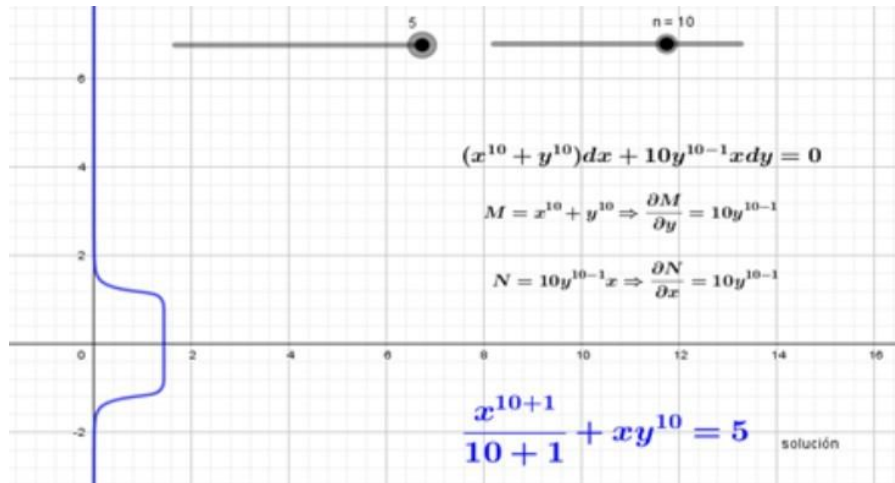


Figure 2

Example 3

Be

$$(ay + b) dx + (ax + c) dy = 0, \quad (2.3)$$

Whose solution is given in blue by $axy + by + cy = k$ in Figure 3, and where a , b , c , and k vary between -5 and 5 .

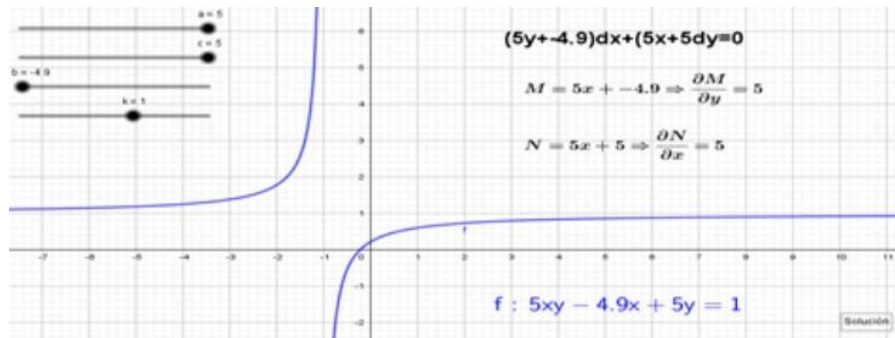


Figure 3

Example 4

Be

$$(y^n e^x - \cos x) dx + n y^{n-1} e^x dy = 0, \quad (2.4)$$

Whose solution is given by $y^n e^x - \sin x = c$ in Figure 4, where c changes between -5 and 5 and n between 2 and 12 .

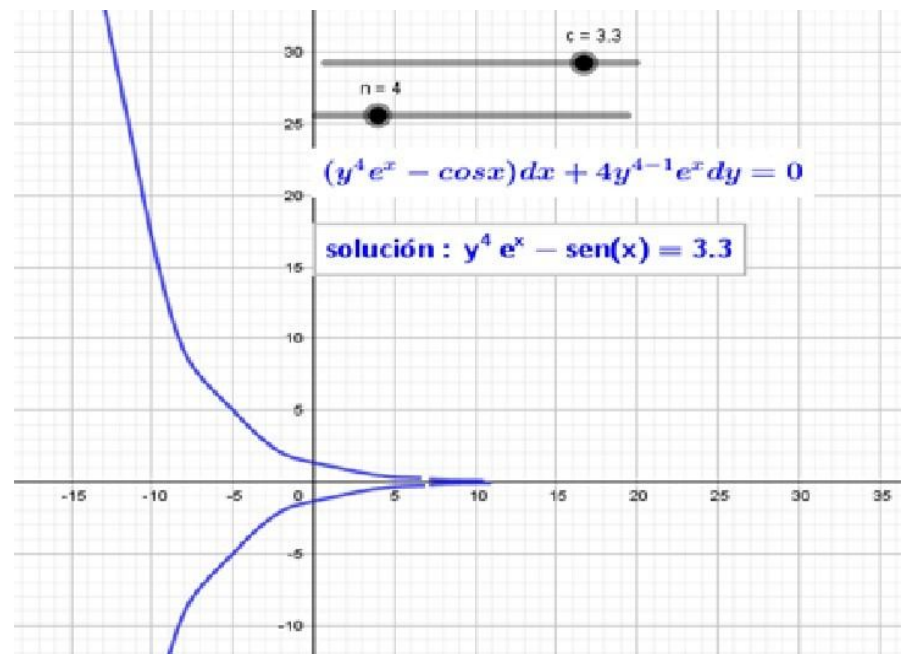


Figure 4

Example 5

Solving the equation of Example 4 by ChatGpt

solve by exact differential equations $(x^3-1)dy+3yx^2dx=0$

<response>

solve by exact differential equations $(x^3-1)dy+3yx^2dx=0$

Answer

To solve the differential equation $(x^3 - 1) dy + 3yx^2 dx = 0$ using the exact method, follow these steps:

1. Identify $M(x, y)$ and $N(x, y)$:

- $M(x, y) = 3yx^2$
- $N(x, y) = x^3 - 1$

2. Check if the equation is exact:

- Compute the partial derivatives:
 - $\frac{\partial M}{\partial y} = 3x^2$
 - $\frac{\partial N}{\partial x} = 3x^2$
- Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, the equation is exact.

3. Find the potential function $\psi(x, y)$:

- Integrate $M(x, y)$ with respect to x :

$$\psi(x, y) = \int 3yx^2 dx = yx^3 + h(y)$$

- Integrate $N(x, y)$ with respect to y :

$$\psi(x, y) = \int (x^3 - 1) dy = x^3y - y + g(x)$$

- Comparing, $h(y) = -y$ and $g(x) = 0$.

4. Write the general solution:

$$\psi(x, y) = x^3y - y = C$$

Simplify to:

$$y(x^3 - 1) = C$$

Where C is the constant of integration.

Figure 5

3 Conclusion

The objective of this paper has been to share applets designed with exact differential equations to help and motivate the learning of mathematics, especially the exact equations in engineering students from Antofagasta and other universities.

References

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